Arrays, Number Puzzles, and Factor Trees

Mathematical goals

This lesson unit is intended to help you assess how well students are able to use "clues" about numbers including: factors, multiples, prime, composite, square, even, odd, etc. In particular, this unit aims to identify and help students who have difficulties with:

- Understanding the difference between primes and composites, or factors and multiples.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards:*

4-OA: **Use the four operations with whole numbers to solve problems.**
4-OA: **Gain familiarity with factors and multiples.**
5-NBT: **Understand the place value system.**
6-NS: **Compute fluently with multi-digit numbers and find common factors and multiples.**

Introduction

This lesson unit is structured in the following way:

• Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their work, and formulate questions for students to answer, to help them improve their solutions.
• During the lesson, students work in pairs and threes to match the array cards, number clues and factor trees.
• In a whole-class discussion, explain their answers.
• Finally, students return to their original assessment task, and try to improve their own responses.

Materials required

Each individual student will need two copies of the worksheet *Arrays, Number Puzzles, and Factor Trees.*
• Each small group of students will need a packet of Card Set 1: Arrays A-J, Number Puzzles 1-8, & Factor Trees a-g; and Card Set 2: Arrays K-T, Number Puzzles 9-16, & Factor Trees h-n copied and cut up before the lesson.

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

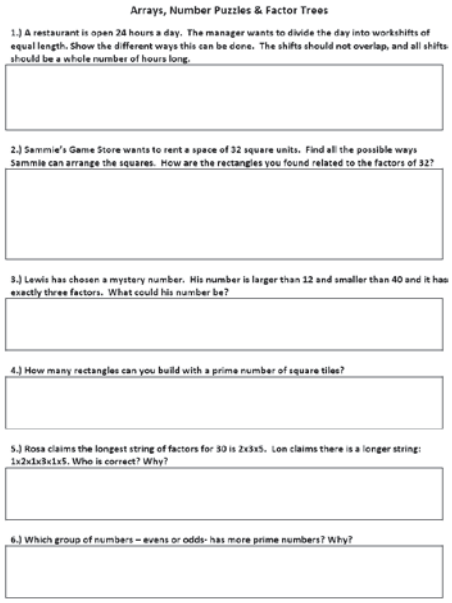
Give each student a copy of *Arrays, Number Puzzles & Factor Trees*. Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering these questions.

Don't worry if you can't figure it out.

There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.



Arrays, Number Puzzles & Factor Trees

1.) A restaurant is open 24 hours a day. The manager wants to divide the day into workshifts of equal length. Show the different ways this can be done. The shifts should not overlap, and all shifts should be a whole number of hours long.

2.) Sammie's Game Store wants to rent a space of 32 square units. Find all the possible ways Sammie can arrange the squares. How are the rectangles you found related to the factors of 32?

3.) Lewis has chosen a mystery number. His number is larger than 12 and smaller than 40 and it has exactly three factors. What could his number be?

4.) How many rectangles can you build with a prime number of square tiles?

5.) Rosa claims the longest string of factors for 30 is 2x3x5. Lon claims there is a longer string: 1x2x1x3x1x5. Who is correct? Why?

6.) Which group of numbers – evens or odds- has more prime numbers? Why?

It is important that students answer the question without assistance, as far as possible.

If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on below.. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Common issues - Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student doesn't find all factor pairs for a given number.	• ***How can you make sure you haven't left out any factor pairs for a number?*** • ***How can you use rectangular arrays to decide if you have all the factor pairs? How can you use a factor tree to decide if you have all the factor pairs? How could you make a chart to decide if you have all the factor pairs?***
Conceptual understanding of prime numbers. Student doesn't realize that a prime number will have only one factor pair, only one rectangular array, and it will not have a factor tree if it is prime. Student should also recognize the only EVEN prime number is 2.	• ***How many other rectangular arrays can you find for each number? What do you notice about the number of rectangular arrays possible for each number? Which number(s) do not have factor tree? Why not?*** • ***Why are more of the numbers even than odd?***
One is neither prime nor composite. Students do not understand that you do not use one as a factor in factor trees.	• ***Why don't any of the factor trees include the number 1 as a factor?***
Not understanding that if there are an odd number of factors for a number, the number is a square number.	• ***Which rectangular arrays are actually squares? How many total factors do each of your numbers have? How can you tell if a number will have a square as one of its arrays?***

Suggested lesson outline

If you have a short lesson, or you find the lesson is progressing at a slower pace than anticipated, then we suggest you end the lesson after the first collaborative activity and continue in a second lesson.

Collaborative activity 1- matching Card Set 1: Arrays A-J, Number Puzzles 1-8, & Factor Trees a-g (30 minutes)

Organize the class into groups of two or three students. With larger groups, some students may not fully engage in the task.

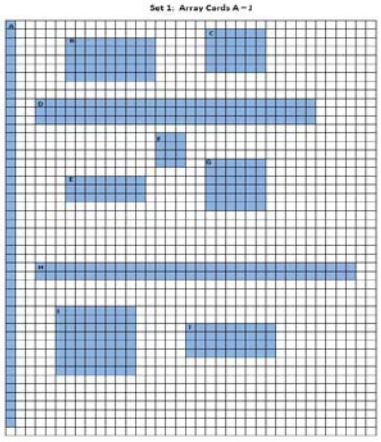
Give each group Card Set 1: Arrays A-J, Number Puzzles 1-8, and Factor Trees a-g.

Introduce the lesson carefully:

I want you to work as a team. Take it in turns to match am Array with either a Number Puzzle or a Factor Tree.

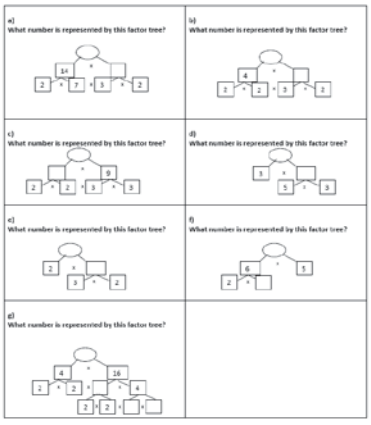
Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.

There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.



Set 1: Number Puzzles 1 – 8

Puzzle 1 1-This number of tiles will make a rectangle 3 tiles wide. 2-This number of tiles will make a rectangle 4 tiles wide. 3- This number is greater than 20. 4- This number is less than 30.	**Puzzle 2** 1-This number is a multiple of 5 and less than 50. 2-This number is not an odd number. 3-This number has exactly 8 factors.
Puzzle 3 1-This number is a factor of 60. 2-This number has 2 digits 3-The sum of the digits is less than 5. 4-One factor of this number is 4.	**Puzzle 4** 1-This number has 2 digits 2-This number is a multiple of 9. 3-This number is even. 4-This number is a square number.
Puzzle 5 1-This number is a multiple of 3 and a multiple of 5. 2-Add the digits in this number and the sum is odd. 3-Multiply the digits and the product is even. 4-This number is odd.	**Puzzle 6** 1-This number divided by 5, has a remainder of 4. 2-This number has 2 digits which are both even. 3-The sum of the digits of this number is 10.
Puzzle 7 1-This number is not an even number, and less than 50. 2-The difference of the digits in this number is 3. 3-This number is not a multiple of 3, 5, or 7. 4- This number is less than 50. 5-The sum of the digits in this number is 11.	**Puzzle 8** 1-The difference between the digits in this number is greater than 3. 2-This number is a multiple of 7, and is not odd. 3-Both digits are even, and the first digit is larger than the second digit. 4-The sum of the digits in this number is great than 10.



Your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may know how to find the number represented by each array or factor tree, but have difficulty finding the answer for each number puzzle.

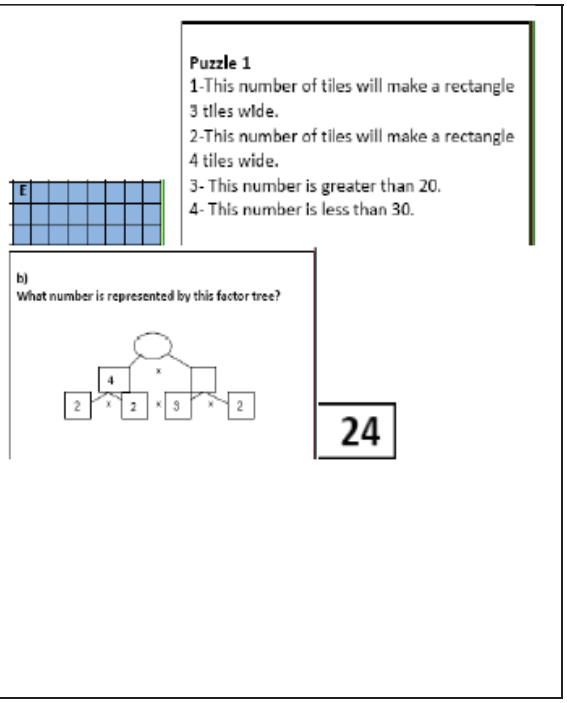
Formative Assessment Lesson Alpha Version

Make a note of student approaches to the task

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors as they find matches for all the cards.

For students struggling to get started:

There is more than one way to tackle this task. Can you think what one of them might be? [Finding the number represented by the rectangular arrays is the most efficient way to find the numbers that will match the number puzzle and factor trees.] *How can you narrow down which numbers might be the solution to each number puzzle?[It may be better to give students who were unable to complete much of the pre-assessment task, only the arrays & factor trees to begin with, then give them the set of number puzzles once they have the list of possible numbers].* *The factor trees are missing numbers, how can you figure out which numbers go in the blanks?* *The use of a hundred's board may be beneficial to some students to "mark off" numbers that do not fit the criteria for the number puzzles. All of the solutions are for numbers of 100 or less.*	

If one student has placed a particular card, challenge their partner to provide an explanation.

Maria placed this Array card with this Number Puzzle. Martin, why has Maria placed it here?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue**,** then you may want to write a couple of questions on the board and organize a whole class discussion.

Collaborative activity 2 - matching Card Set 2:

Arrays K-T, Number Puzzles 9-16, & Factor Trees h-n (30 minutes)

Formative Assessment Lesson Alpha Version

Card Set 2 can be used in the exact same way as card set one. Set 2 does include clues using the exponents which are not included in set 1. You may have students work on set one the first day and set two a second day, or have all groups of students use card set one and only have card 2 as an **extension activity** for who finish the first card set quickly. If you have all students complete both sets, an **Extension Activity** on either day would be to have students **write their own number puzzles and make corresponding arrays and factor trees** to match the puzzles they have made. Groups who finish making their own number puzzles can switch creations with other groups to test out the new puzzles.

Wrap up the lesson when students are completely satisfied with their own work by them glue or tape their cards together in solution sets including arrays, puzzles & factor trees that match. They should have eight matching solution sets for Card Set 1 and another eight for Card Set 2.

Sharing work (10 minutes)

When students get as far as they can with matching cards, ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

> *If you are staying at your desk, be ready to explain the reasons for your group's matches.*
>
> *If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.*
>
> *If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.*
>
> *When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.*

Students may now want to make changes.

Taking two lessons to complete second card sort

The second card set, will be completed after Card Set 1 has been taped or glued on a large sheet of paper.

Have students make up their own number puzzles as an extension n either day as needed for an extension.

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Arrays, Number Puzzles & Factor Trees*, as well as a second blank copy of the task.

> *Look at your original responses and think about what you have learned this lesson.*
>
> *Using what you have learned, try to improve your work.*

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Solutions

ANSWER KEY SET 1

Array Card	Number Puzzle	Factor Tree	Number
E	1	b	24
C	2	f	30
F	3	e	12
G, J	4	c	36
B	5	d	45
H, I	6	g	64
A	7	--	47
D	8	a	84

ANSWER KEY SET 2

Array Card	Number Puzzle	Factor Tree	Number
R	9	k	70
D, T	10	l	28
K	11	--	17
N, P	12	m	20
S	13	h	50
L	14	n	16
O	15	i	100
M	16	j	75

Assessment Task: ***Arrays, Number Puzzles & Factor Trees***

For questions 1 and 2 student responses will vary depending on how they solve multiplication problems. Be sure that the responses have correct interpretations of each model.

Question 1:

# of shifts	1	2	3	4	6	8	12	24
# of hours	24	12	8	6	4	3	2	1

Question 2: The dimensions of the sides of the rectangles should be the factors for 32. Students should drawn rectangles with the following dimensions:

1x32, 2x16, 4x8 (or the reverse 32x1, 16x2, 8x4)

Question 3: Numbers with exactly three factors would be square numbers. Square numbers between 12 and 40 are: 16, 25, & 36. Only 25 has only three factors.

Question 4: only one because prime numbers only have two factors – one and itself.

For example 7 is prime and has only factors 1 and 7 so the only array would be 1x7 (or 7x1).

Question 5: Rosa is correct. Strings of factors at the bottom of a factor tree do not include 1.

Question 6: All prime numbers are odd, except for the number 2 so ODDS have more prime numbers.

These materials were adapted from *Connected Mathematics: Prime Time,* and *Investigations in Number Data & Space: Mathematical Thinking.*

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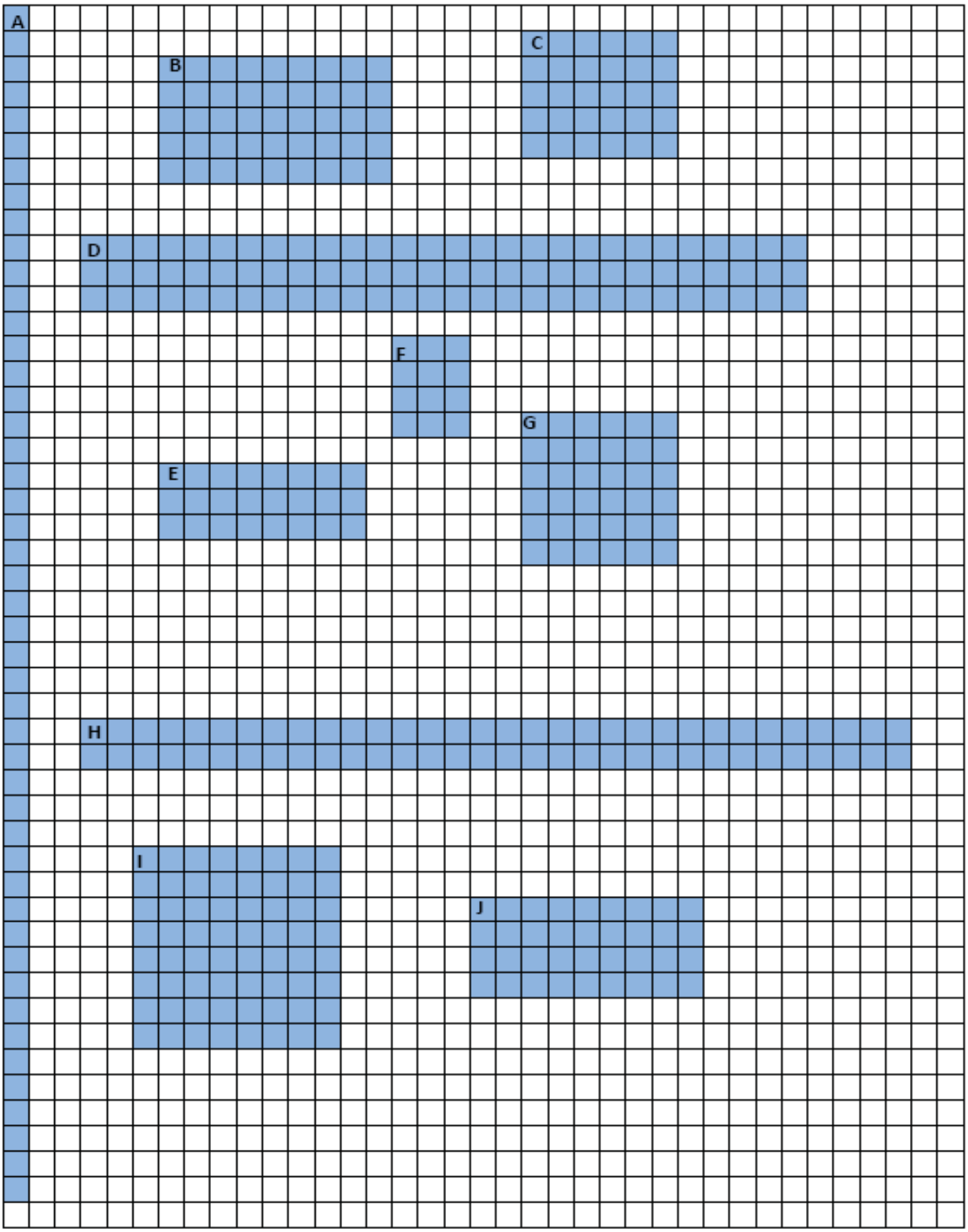
Set 1: Array Cards A – J



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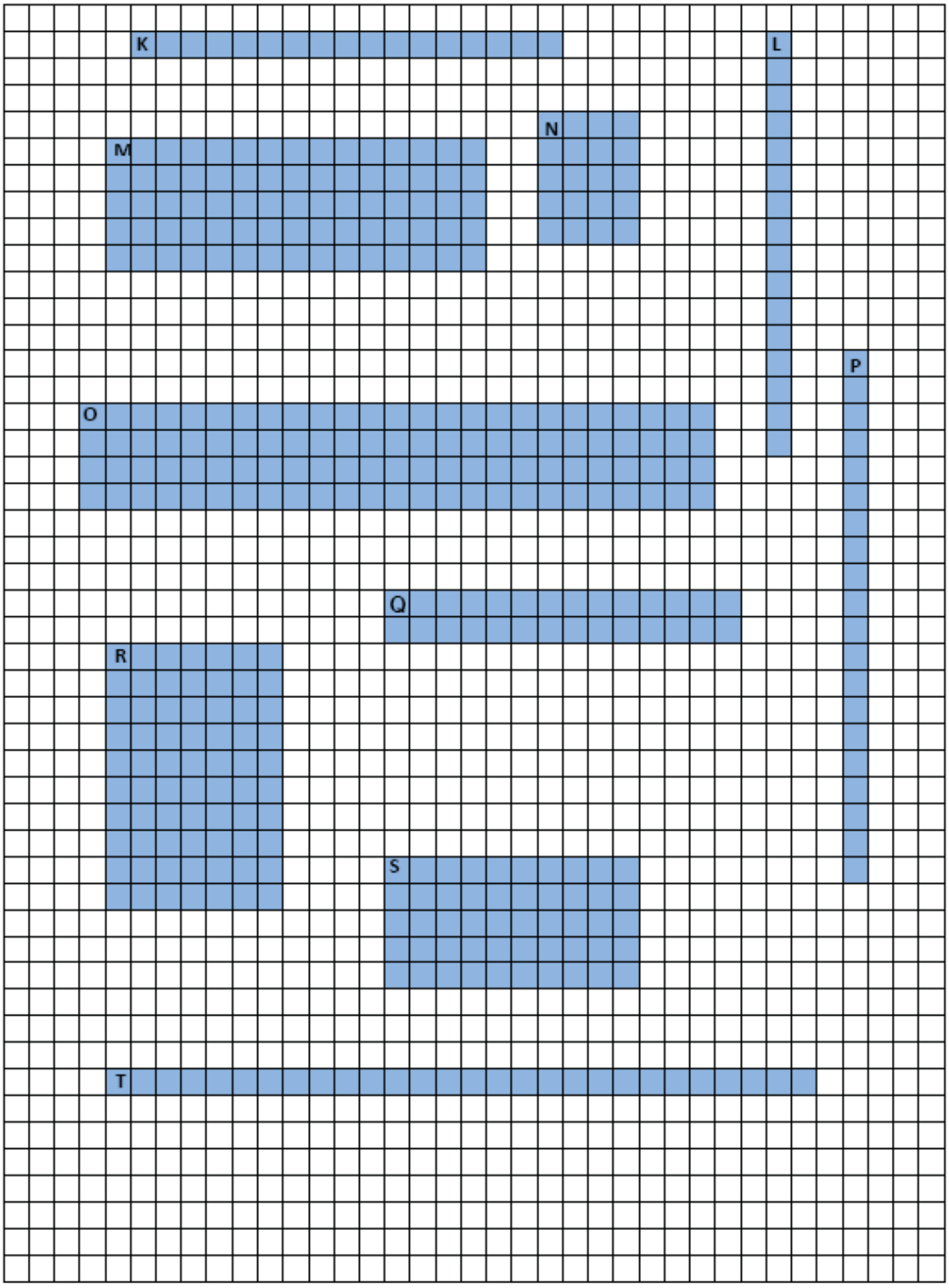
Set 2: Array Cards K – T

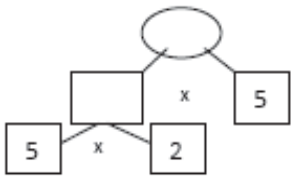


Set 2: Number Puzzles 9 – 16

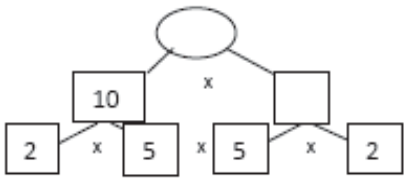
Puzzle 9 1-This number is a multiple of 2 and 7. 2-This number is less than 100 and larger than 50. 3-This number is the product of 3 different primes.	**Puzzle 10** 1-This number has two digits, and has 4 as a factor. 2-The sum of the digits in this number is 10. 3-The difference between the digits is 6.
Puzzle 11 1-This number is less than 20. 2-The sum of the digits in this number is greater than 7. 3-This number is prime.	**Puzzle 12** 1-This number is a factor of 10^3. 2-This number is less than 45 and greater than 12. 3-This number is a multiple of 4.
Puzzle 13 1-This number is a multiple of 5, but does not end in 5. 2-This number is bigger than the 7^{th} square number but smaller than the 9^{th} square number. 3-Two of the three numbers in the prime factorization of this number are the same.	**Puzzle 14** 1-The sum of the two digits in this number is odd. 2-The only prime number in this number's prime factorization is 2. 3-This number of tiles will make a square.
Puzzle 15 1-This number is a composite number. 2-This number is a factor of 10^3, and 10^2. 3-This number has exactly 9 factors. 4-The sum of the digits in this number is neither prime nor composite.	**Puzzle 16** 1-This number is a multiple of 3 and a multiple of 5. 2-The sum of the digits is even. 3-This number is the largest number less than 100 that fits all the other clues. 4-When you multiply the digits of this number you get an odd number for the product.

Set 2: Factor Trees h – n

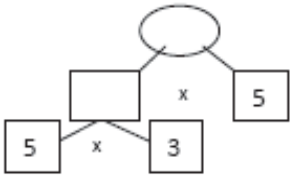
h)
What number is represented by this factor tree?



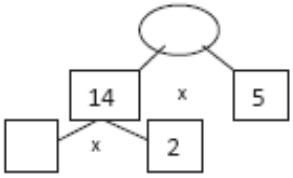
i)
What number is represented by this factor tree?



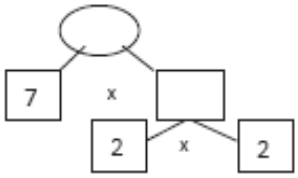
j)
What number is represented by this factor tree?



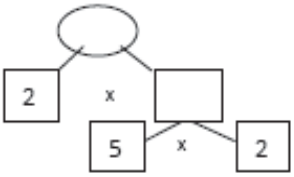
k)
What number is represented by this factor tree?



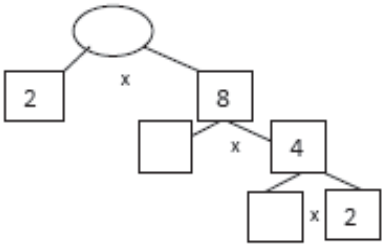
l)
What number is represented by this factor tree?



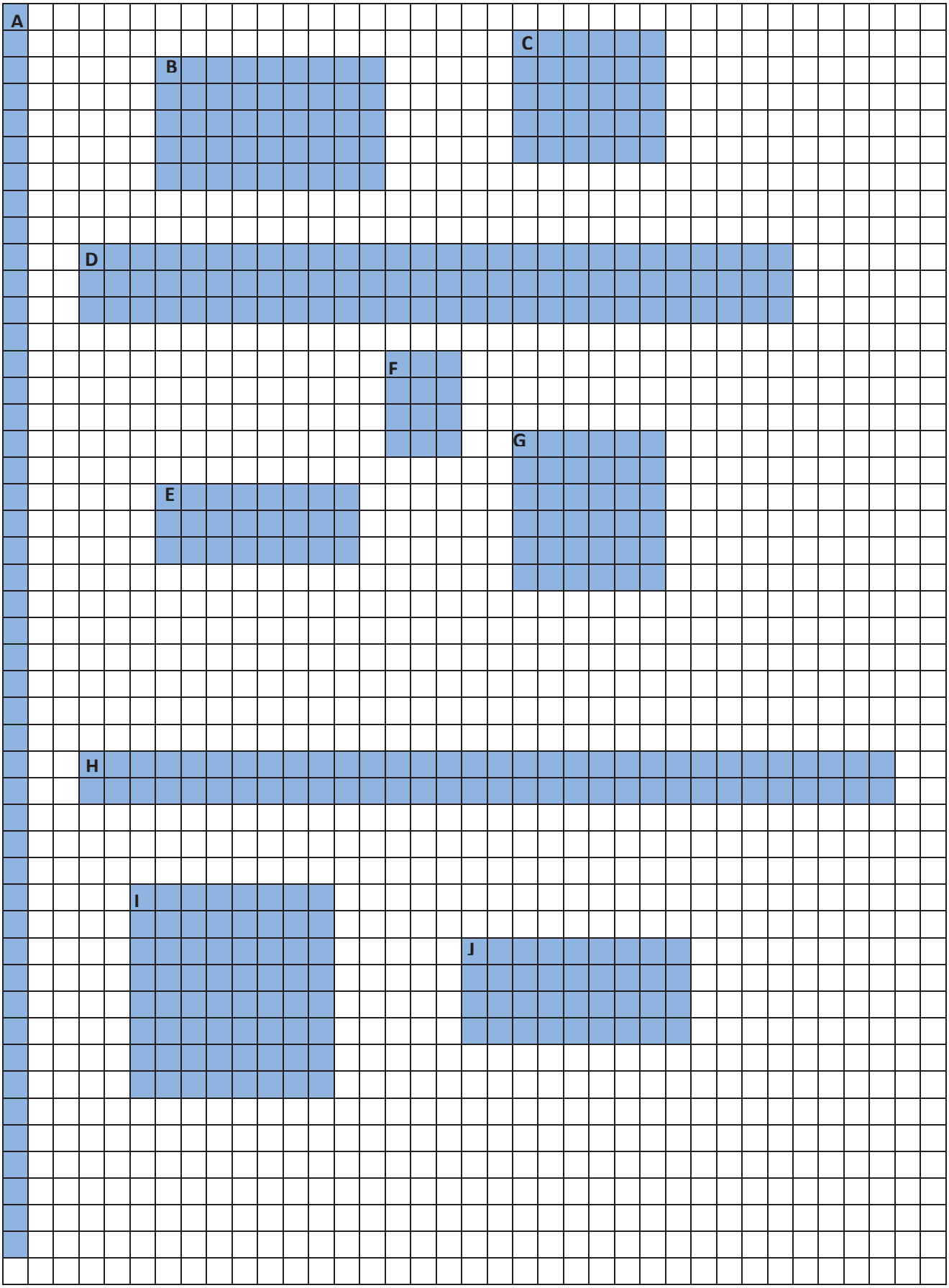
m)
What number is represented by this factor tree?



n)
What number is represented by this factor tree?



Set 1: Array Cards A – J



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Set 1: Factor Trees a - g

a)
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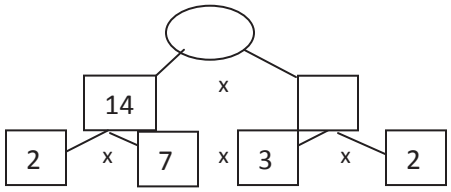
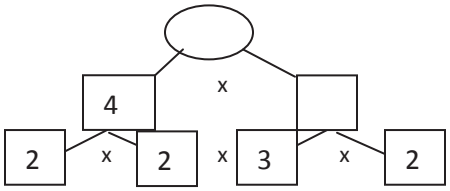
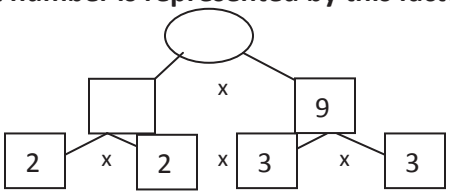


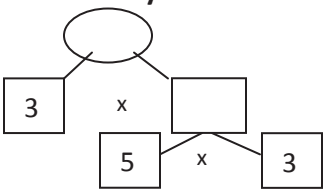
b)
What number is represented by this factor tree?



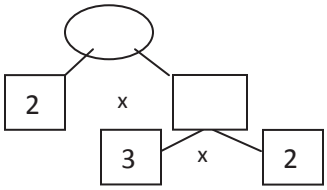
c)
What number is represented by this factor tree?



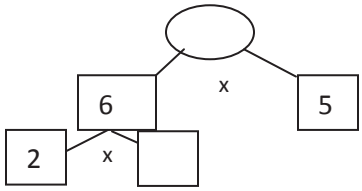
d)
What number is represented by this factor tree?



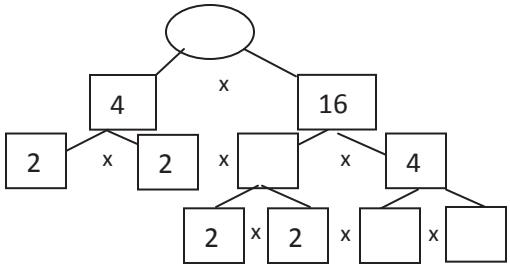
e)
What number is represented by this factor tree?



f)
What number is represented by this factor tree?



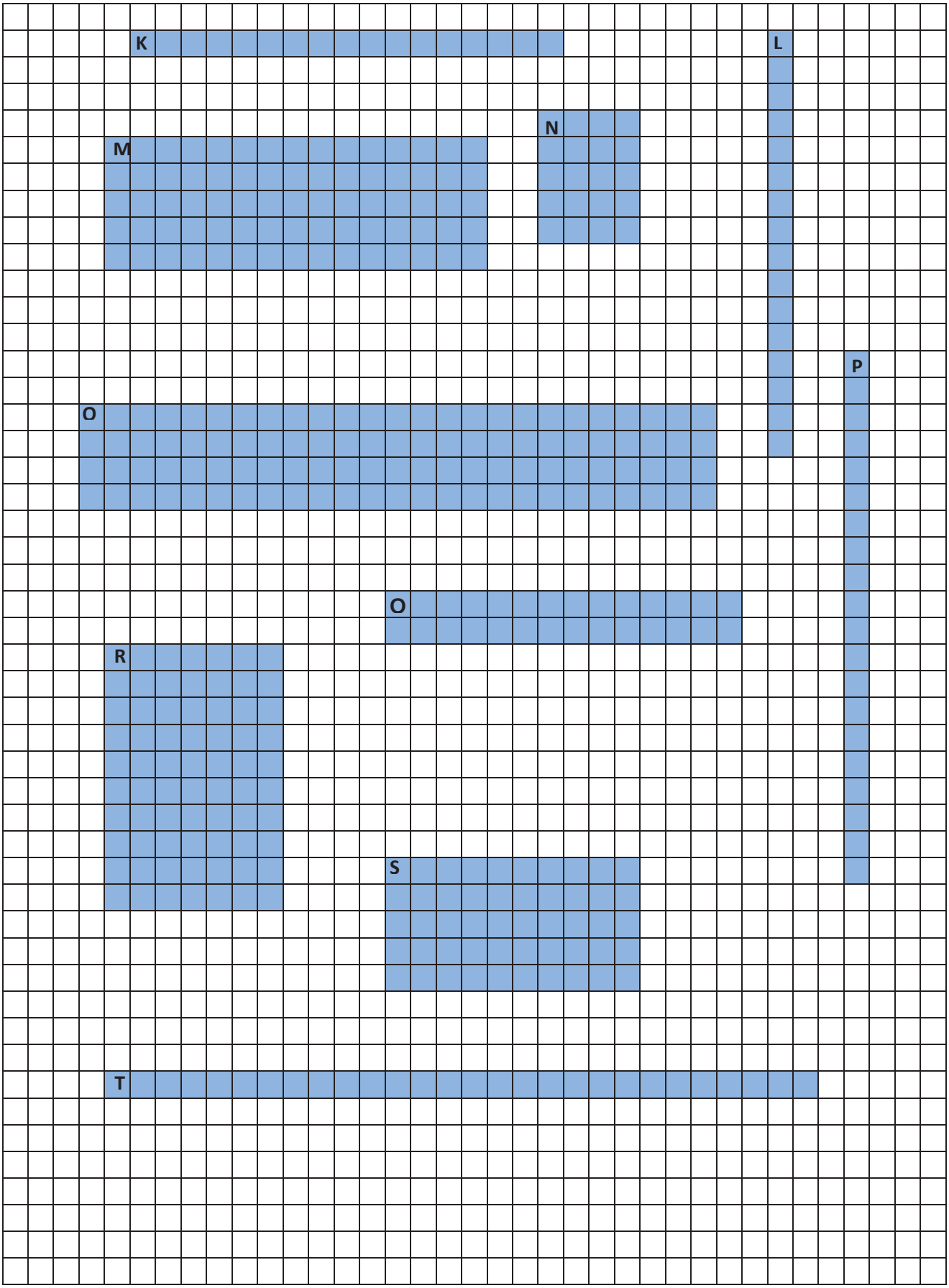
g)
What number is represented by this factor tree?



ANSWER KEY SET 1

Array Card	Number Puzzle	Factor Tree	Number
E	1	b	24
C	2	f	30
F	3	e	12
G, J	4	c	36
B	5	d	45
H, I	6	g	64
A	7	--	47
D	8	a	84

Set 2: Array Cards K – T

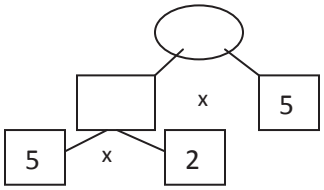


Set 2: Number Puzzles 9 – 16

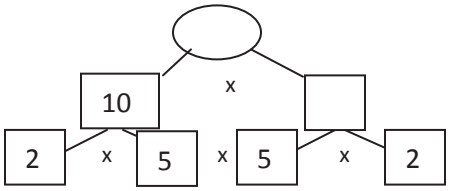
Puzzle 9 1-This number is a multiple of 2 and 7. 2-This number is less than 100 and larger than 50. 3-This number is the product of 3 different primes.	**Puzzle 10** 1-This number has two digits, and has 4 as a factor. 2-The sum of the digits in this number is 10. 3-The difference between the digits is 6.
Puzzle 11 1-This number is less than 20. 2-The sum of the digits in this number is greater than 7. 3-This number is prime.	**Puzzle 12** 1-This number is a factor of 10^3. 2-This number is less than 45 and greater than 12. 3-This number is a multiple of 4.
Puzzle 13 1-This number is a multiple of 5, but does not end in 5. 2-This number is bigger than the 7^{th} square number but smaller than the 9^{th} square number. 3-Two of the three numbers in the prime factorization of this number are the same.	**Puzzle 14** 1-The sum of the two digits in this number is odd. 2-The only prime number in this number's prime factorization is 2. 3-This number of tiles will make a square.
Puzzle 15 1-This number is a composite number. 2-This number is a factor of 10^3, and 10^2. 3-This number has exactly 9 factors. 4-The sum of the digits in this number is neither prime nor composite.	**Puzzle 16** 1-This number is a multiple of 3 and a multiple of 5. 2-The sum of the digits is even. 3-This number is the largest number less than 100 that fits all the other clues. 4-When you multiply the digits of this number you get an odd number for the product.

Set 2: Factor Trees h – n

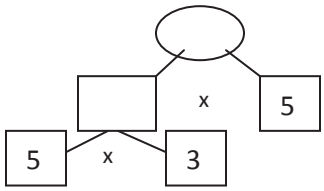
h)
What number is represented by this factor tree?



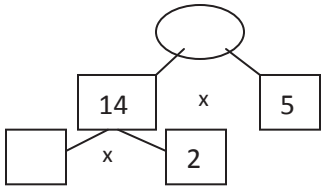
i)
What number is represented by this factor tree?



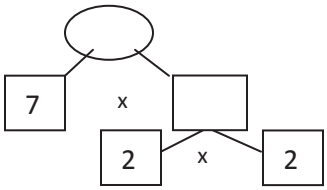
j)
What number is represented by this factor tree?



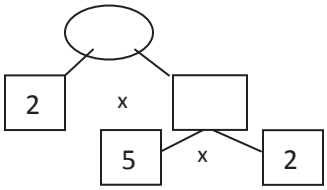
k)
What number is represented by this factor tree?



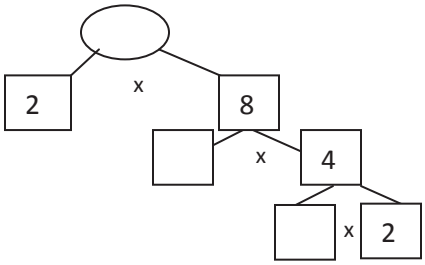
l)
What number is represented by this factor tree?



m)
What number is represented by this factor tree?



n)
What number is represented by this factor tree?



ANSWER KEY SET 2

Array Card	Number Puzzle	Factor Tree	Number
R	9	k	70
D, T	10	l	28
K	11	--	17
N, P	12	m	20
S	13	h	50
L	14	n	16
O	15	i	100
M	16	j	75