Relating Fractions Equivalencies to Decimal Fractions

Mathematical goals
This lesson unit is intended to help you assess how well students are able identify equivalent decimal fractions.

Students will
- Recognize and generate equivalent fractions.
- Use equivalent fractions to add and subtract fractions with like denominators.
- Use decimal notation for fractions with denominators 10 and 100.
- Use words to indicate the value of the decimal.
- Use decimal fractions and locating them on the number line.
- Use area models to represent equivalent fractions and decimals.

Common Core State Standards
This lesson involves mathematical content in the standards from across the grade, with emphasis on:

4.NF
- Extend understanding of fraction equivalence and ordering. (Note: Ordering of fractions is not addressed in this lesson.)
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

This lesson involves a range of Standards for Mathematical Practice with emphasis on:
1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Introduction
This lesson unit is structured in the following way:
- A day or so before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. Then, you review their work and formulate questions for students to answer to help them improve their solutions.
- During the lesson, students work in pairs to match the fraction and addition problems with fraction and decimal equivalencies, the correct number line that represents the fraction/decimal, and an area model representation.
- In a whole-class discussion, students will justify their answers.
- You may choose to have students revisit the original assessment task, and try to improve their own responses.
Materials required

- Each student will need 2 copies of the assessment to use a pre-assessment and a revisit.
- Each pair of students, during the collaborative lesson, will need a packet of Card Set A – G. (Start with Card Sets A and B. After students can demonstrate their reasoning for the matches, give them the next ‘layer’ of cards. You may want to make copies of the card sets on different color card stock to assist with organization.
- The card sets should be cut up before the lesson.

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Frame the lesson:
Explain to the students the purpose of, your expectations for and the timeline of the lesson that will follow across the next several days.

Assessment task:
Have the students do this task in class a day or more before the Formative Assessment (collaborative) Lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of Pre-Assessment. Introduce the task briefly help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering these questions.
Don’t worry if you can’t figure it out.
There will be a lesson on this material [tomorrow] that will help you improve your work and an opportunity to show how much your understanding has grown.

It is important that students answer the question without assistance, as far as possible. If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them and be conscientious to not lead or provide the thinking for your students.

Assessing students’ responses

Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.
We suggest that you do not score students’ work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on below. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your students’ work, using the ideas below. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

**Common Issues - Suggested questions and prompts:**

<table>
<thead>
<tr>
<th>Common Issues</th>
<th>Suggested questions and prompts</th>
</tr>
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</table>
| Students use the idea of (# shaded) divided by (#total), but cannot find an equivalent fraction. *(Question 1)* | • *Can you think of a smaller number of total parts than 100 to represent this whole? (10 parts..so 2/10)*  
  • *How many rectangles, of the same size of the shaded part, are there in the whole? (5..so 1/5 of the whole is shaded)* |
| Students incorrectly identify fractional (or decimal) representations on the number line, perhaps by identifying the next missing part as the next number in the pattern, without considering the parts that had been left unidentified. *(Question 2)* | • *How can you tell the number of equal divisions there are between 0 and 1 on the number line?*  
  • *Can you find ½ on the number line? (anchor fraction)* |
| Students mis-apply an algorithm without having understanding of what it means to add fractions (conceptually). Each part of the fraction (numerator/denominator) is treated as a different single-digit whole number. *(Question 3)* | • *What is one-tenth plus one-tenth? (This question builds on 3rd grade standard of using unit fractions to accumulate.)* |
Suggested lesson outline

Whole-class interactive introduction (10 minutes)

Give each student a mini-whiteboard, a marker, and an eraser.

Explain to the class that in the lesson they will be working with fractions and decimals and locating them on a number line.

Ask students to write on their mini-whiteboards the answers to questions such as the following. Each time, ask students to explain their method.

“Write a fraction which is equivalent to ¾” – ask a few students to explain how they know their fraction is equivalent.

“Write a decimal which is equivalent to 7/10” – ask a student to explain how they did this.

“Draw a number line to compare 2/5 and 3/10” – ask several students to explain their comparison.

Collaborative activity: Matching Expressions, Number lines and Area models (15 minutes)

Organize the class into groups of two or three students. With larger groups, some students may not fully engage in the task. Display the slide “Matching expressions, number lines and areas.” Note that the examples provided are not actual matches. The purpose of the slide is to illustrate the ideas and provide instructions for the collaborative work. Leave these instructions visible for students for the duration of the activity.

I am going to give you three sets of cards to match, one with expressions, one with a value on a number line and one with a value expressed as an area.

Take turns with your partner matching an expression with a number line or area model. Place the cards side by side on the table and explain your reasoning. It is your partner’s job to agree with or question your choices.

Give each group Card Set A: addition/subtraction, Card Set F and Card Set G.

Explain to students how they should work together, making sure that each student can articulate why the card is placed where it is, even if that student didn’t place the card.

While students are working, you have two tasks: to find out about students’ work and to support their reasoning.

Find out about students’ work – circulate, listen, take notes, keep groups advancing through card sets
As you move around the room listen to students’ explanations.
Your tasks during the small group work are to make a note of student approaches to the task, to support student problem solving and to monitor progress. Note any difficulties that emerge for more than one group; these can be discussed later in the lesson.

Collaborative Activity: matching expressions, number line and area representations, decimals and words

Provide each group with Card Set D and E. The students should match these expressions with the sets from the first part of the Collaborative Activity.

Give each group a poster and glue stick and provide these instructions.

Match these cards to those already on your table.

When your whole group agrees, paste down your final arrangement onto a poster. Next to each group, write down how you know that the decimal values express the sum or difference in the expression.

Whole Class Interactive Discussion/Plenary (15 minutes)

Hold a whole class interactive discussion where you ask groups to justify, using their posters, why the sum/difference expressions and decimal values are equivalent. This is the students’ opportunity to share what they have learned and to learn from each other’s reasoning.

Then, use the mini-whiteboards and questioning to begin to generalize the learning.

Draw a number line and mark where 1/5 would lie.

Write another fraction that shows the same place on the number line.

Draw an area model that shows a value of 3/10ths.

Write another fraction that could describe the same area.

What is the sum of 2/5 + 1/5.

Write another fraction or a decimal that is equivalent to your answer.

Administer the post-assessment (15 min)

Post or pass out the feedback questions you wrote from the students’ pre-assessments. Also, pass back their pre-assessments and a clean copy of the post-assessment. Direct students to read through their work and think about what they have learned, then to write a new solution to see how much they can improve their work.
### CARD SET A

<table>
<thead>
<tr>
<th>A1</th>
<th>[\frac{2}{10} + \frac{3}{10}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>[\frac{8}{10} + \frac{2}{10}]</td>
</tr>
<tr>
<td>A3</td>
<td>[\frac{1}{5} + \frac{1}{5}]</td>
</tr>
<tr>
<td>A4</td>
<td>[\frac{9}{5} - \frac{2}{5}]</td>
</tr>
<tr>
<td>A5</td>
<td>[\frac{1}{10} + \frac{1}{10}]</td>
</tr>
<tr>
<td>A6</td>
<td>[\frac{10}{10} - \frac{4}{10}]</td>
</tr>
<tr>
<td>A7</td>
<td>[\frac{10}{5} - \frac{6}{5}]</td>
</tr>
<tr>
<td>A8</td>
<td>[\frac{148}{100} - \frac{38}{100}]</td>
</tr>
<tr>
<td>A9</td>
<td>[\frac{72}{100} - \frac{42}{100}]</td>
</tr>
<tr>
<td>A10</td>
<td>[\frac{17}{100} + \frac{53}{100}]</td>
</tr>
</tbody>
</table>
## CARD SET D

<table>
<thead>
<tr>
<th>D1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>0.6</td>
</tr>
<tr>
<td>D3</td>
<td>0.8</td>
</tr>
<tr>
<td>D4</td>
<td>1.0</td>
</tr>
<tr>
<td>D5</td>
<td>0.5</td>
</tr>
<tr>
<td>D6</td>
<td>1.1</td>
</tr>
<tr>
<td>D7</td>
<td>0.4</td>
</tr>
<tr>
<td>D8</td>
<td>1.4</td>
</tr>
<tr>
<td>D9</td>
<td>0.3</td>
</tr>
<tr>
<td>D10</td>
<td>0.7</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>E1</td>
<td>five-tenths</td>
</tr>
<tr>
<td>E2</td>
<td>one</td>
</tr>
<tr>
<td>E3</td>
<td>two-tenths</td>
</tr>
<tr>
<td>E4</td>
<td>one and four-tenths</td>
</tr>
<tr>
<td>E5</td>
<td>three-tenths</td>
</tr>
<tr>
<td>E6</td>
<td>six-tenths</td>
</tr>
<tr>
<td>E7</td>
<td>eight-tenths</td>
</tr>
<tr>
<td>E8</td>
<td>one and one-tenth</td>
</tr>
<tr>
<td>E9</td>
<td>four-tenths</td>
</tr>
<tr>
<td>E1</td>
<td>seven-tenths</td>
</tr>
</tbody>
</table>
CARD SET F

F2

F1

F4

F3

F6

F5

F8

F7

F10

F9
1. Circle every value below that is equivalent to $\frac{3}{5} + \frac{1}{5}$.

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<tbody>
<tr>
<td>$\frac{4}{10}$</td>
<td>0.8</td>
<td>$\frac{20}{5}$</td>
<td>$\frac{4}{5}$</td>
<td>4.0</td>
<td>1.25</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Explain your reasoning below.

Fill in the missing parts on the number line.

Add/Subtract the Fractions | Equivalent Fraction for the solution: | Decimal equivalence to the solution:
--- | --- | ---
$\frac{1}{10} + \frac{7}{10}$ | $\frac{8}{10}$ | 0.8
$\frac{3}{5} + \frac{4}{5}$ | $\frac{7}{5}$ | $\frac{14}{10}$
$\frac{75}{100} - \frac{45}{100}$ | $\frac{30}{100}$ | 0.3